$\delta(n)$ = Dirac delta function $\int_{-\infty}^{\infty} \delta(n)\,dn = 1$
H = holdup per stage
k = slope of pseudo equilibrium curve
K_1, K = tuning parameter
L = liquid flow rate
$\mathcal{L}$ = linear operator
$\mathcal{L}^*$ = adjoint of $\mathcal{L}$
λ_i = ith eigenvalue of $\mathcal{L}_n^* + \mathcal{L}_m^*$
M = total number stages
ρ = a parameter = $\dfrac{\bar{L} - k\bar{V}}{2k\bar{V}}$
S = matrix of scaling factors, $s_{ii} = 1/s_i : S_{ij} = 0$
T_n = temperature of liquid stream x_n
x_n = liquid composition on the nth plate
y_n = vapor composition passing liquid stream x_n
V = vapor flow rate

A bar over a quantity indicates that it takes on only steady state values.

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Gas-Liquid Slug Flow with Drag-Reducing Polymer Solutions

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The two-phase slug flow regime in horizontal cocurrent gas-liquid flow was studied for the case of the liquid phase containing small amounts of polyacrylamide, a drag-reducing long chain polymer. The experimental work was performed in a 2.54 cm I.D. horizontal test section 10.7 m long.

Results indicated that two-phase drag reduction was greater than in single phase flow at the same superficial liquid velocities. By the use of pressure drop results with and without polymer additive, and a knowledge of slug geometry, frequency, and velocity, it was possible to separate approximately the contribution to the pressure drop of liquid wall friction and slug inertial effects. In all cases, the acceleration term was important and is the major energy term at most flow conditions. The technique may be of general usefulness in determining accelerational effects in two phase flows.

When drag reducing agents have been added to a horizontal slug flow, the pressure loss can be correlated successfully both by the Lockhart-Martinelli type of relationship or by a universal drag reduction curve of the type proposed by Virk.

Concurrent gas-liquid flow is frequently encountered in engineering processes. It occurs in boiler tubes, distillation columns, in polymer processing, and in chemical reactor applications. This type of flow has many unique features, and these must be evaluated in each situation. However, one phenomenon which is nearly always undesirable is the high axial pressure gradient, with a resultant substantial energy consumption per unit volume of liquid throughput.

It was observed by Toms (*1*) in 1948 that a substantial reduction of the frictional pressure gradient in one-phase turbulent flow could be achieved by the addition of dilute long chain polymers in solution, an effect now

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sometimes called the Toms Phenomenon. At the present time, no complete explanation exists for this behavior, and recent work has not yet been able to distinguish between proposed old and new theories. Experimental evidence shows that the polymer has the effect of increasing the thickness of the viscous sublayer and transition zone. The mechanism of this boundary layer effect is not yet fully understood, but supporting experimental evidence has been given by Arunachalam (*2*), Fortuna and Hanratty (*3*), and Rudd (*4*).

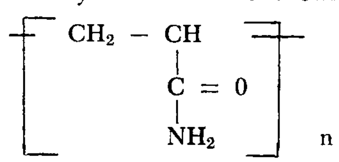
Because of the industrial importance of gas-liquid flow and the associated problem of large axial pressure gradients, a program was undertaken to investigate the behavior of a two-phase system containing a drag-reducing polymer solution.

The present study involves the slug flow regime. In this regime, the interface waves have such large amplitudes that they completely bridge the tube cross section. The slug of liquid thus formed is then accelerated through the tube by the force of the gas buildup behind the slug.

Slug flow was studied for two reasons. Firstly, it is an important flow regime as far as transport processes are concerned. Rosehart (*5*) has demonstrated that for a two-phase chemical reactor, optimal operating conditions are often in the slug flow regime. Gorman (*6*), investigating wall pressure fluctuations, and St. Pierre (*7*), investigating turbulent interchange mixing, have also observed maxima of the phenomena under investigation in slug flow. Secondly, because of the unusual nature of slug flow, that is, alternating sections of gas and liquid, it is possible to use single phase drag reduction information combined with two-phase data to suggest an analysis of the physical structure of slug flow.

In the present study, Polyhall 295 a polyacrylamide polymer distributed by Stein Hall Canada Ltd. (*8*) has been utilized. This particular polymer was used because it is commercially available and easy to use, and it requires relatively low concentrations in water (< 200 ppm) and the solution density remains the same as that of water. A check of the static surface tension produced results which were also the same as water.

The fact that small additions of polymer did not appear to affect the bulk physical properties of water allowed for a clearer interpretation of the results. Another characteristic of Polyhall which made it attractive for the present work was a greater than usual resistance of Polyhall 295 to shear degradation. Polyhall 295 has the following structure:



The average molecular weight is in the range 1-6 $\times$ 10^6.

EXPERIMENTAL CONDITIONS

A low pressure isothermal concurrent gas-liquid flow apparatus was used in the experimental work. The test section of polycarbonate tubing was 1-in. I.D. and approximately 10.7 m long. A diagram of the flow loop is given in Figure 1. The air flow was measured by rotameters while the liquid phase flow was monitored by a magnetic flow meter with dial read out. The calibration of the liquid flow rate was found to be independent of the viscosity of the liquid. A Moyno progressive-cavity pump was used to circulate the liquid phase.

To determine the axial pressure gradient, a liquid-filled manometer system was used with carbon tetrachloride as the manometer fluid. The following special precautions were taken to time-average the pressure oscillations so characteristic of slug flow:

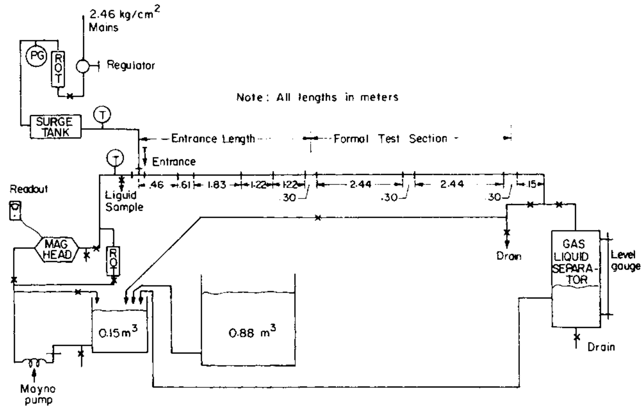


Fig. 1. Experimental flow loop.

1. Use of a radial pressure ring (*9*).
2. Large liquid reservoirs before the manometer.
3. Use of short lengths of capillary tube before the manometer to dampen the oscillations.
4. Use of liquid filled manometer lines (high inertia).

The flow loop was equipped also with a strain gauge transducer system to monitor the pressure gradient, but the low gradients observed with the polymer solution were outside the useful range of the transducers. However, later work with concentrated polymer solutions indicated close agreement between the manometer pressure gradients and those obtained from the transducer system.

Slug velocities and frequencies were determined by visual observations. Other slug properties were taken from high speed traces of the void fraction using an electrical void meter (*10*) developed during this study. This meter, of the induction type, was capable of frequency responses of better than 10^4 s^{-1}.

THEORY

Slug Flow Model for Pressure Loss

Based on careful visual and photographic observations, Hubbard and Dukler (*11*) have postulated that a scooping mechanism is the characteristic behavior of slug flow. According to this mechanism, the liquid slug is continuously picking up liquid at the nose of the slug, accelerating this fluid to the slug velocity, and discharging the same amount of liquid at the tail of the slug (steady state). The pick-up and accceleration of the fluid in front of the fast moving slug makes a contribution to the axial pressure gradient which is sometimes called the accelerational term. For the present study, the following assumptions have been made regarding slug flow:

1. The average liquid velocity in the liquid slug head is equal to the no-slip velocity, which is defined as the sum of the superficial gas and liquid velocities [as per Hubbard and Dukler (*11*)]. Superficial velocity is the conduit velocity calculated as if only that phase was flowing at the specified volumetric rate for that phase.
2. The liquid flowing in the tail of the liquid slug moves at a very low velocity. In fact, under certain conditions the fluid appears to be nearly motionless (this assumption is based on photographic and visual observation).
3. The mechanism of drag reduction in the head of the slug is the same as for one-phase turbulent flow, and there is no effect in the tail of the slug, which will be in laminar flow.
4. The slug moves through the conduit at a frequency of γ_S, slug/s and at a translational velocity of V_S, cm/s. The axial pressure drop is assumed to be only a minor percentage of the average system absolute pressure.

Based on these assumptions, the axial pressure drop across one slug "unit" in a horizontal tube can be expressed as the sum of two contributions

$$\Delta P_S = \Delta P_{LE} + \Delta P_f \tag{1}$$

where ΔP_{LE} is the component of the pressure gradient due to acceleration effects or scooping, and ΔP_f is the frictional component of the axial pressure gradient. The frictional loss, which is generated primarily by the high velocity liquid slug head (see Figure 2), can be expressed as follows assuming that only this contribution is important, and ignoring gas phase frictional effects or those in the tail of a slug

$$\Delta P_f = \frac{2 f_s \rho_L V_{ns}^2 R_s l_s}{D g_c} \tag{2}$$

where $f_s = 0.0014 + 0.125/(N_{Re,s})^{0.32}$

$$N_{Re,s} = \frac{D V_{ns} \rho_L}{\mu_L}$$

ρ_L = liquid phase density

$V_{ns} = V_{SL} + V_{SG}$ = no-slip velocity

l_s = length of liquid slug head

Based on Equations (1) and (2) an expression can be deduced for the slug flow pressure gradient

$$\left(\frac{\Delta P}{\Delta L}\right)_{2\phi} = \left[\frac{\Delta P_{LE}}{l_t} + \frac{2 f_s \rho_L V_{ns}^2 R_s l_s}{D g_c\, l_t}\right] \frac{\Delta L\, \gamma_s}{V_s} \tag{3}$$

where V_s is the slug translational velocity

ΔL = test section length

l_t = total length of one slug unit

If Equation (3) is now applied to identical slug flow cases, but one having polymer additive and one with no polymer added, then a ratio can be taken as in Equation (4)

$$\frac{\left(\frac{\Delta P}{\Delta L}\right) 2\phi_{wp}}{\left(\frac{\Delta P}{\Delta L}\right) 2\phi_{np}} = \frac{\frac{\Delta P_{LE}}{l_s} + \frac{2 f_{swp}\, \rho_L V_{ns}^2 R_s}{D g_c}}{\frac{\Delta P_{LE}}{l_s} + \frac{2 f_{snp}\, \rho_L V_{ns}^2 R_s}{D g_c}} \tag{4}$$

where the subscript wp means with polymer additive, and subscript np indicates no polymer additive.

If the effect of the polymer in reducing drag is known from the single phase case, then in principle Equation (4) can be solved for $\Delta P_{LE}/l_t$ the accelerational or lost energy component of the two-phase axial pressure gradient. Four separate experiments would be needed, two single-phase and two gas-liquid flow tests. This approach was not satisfactory because it was difficult to maintain the same test

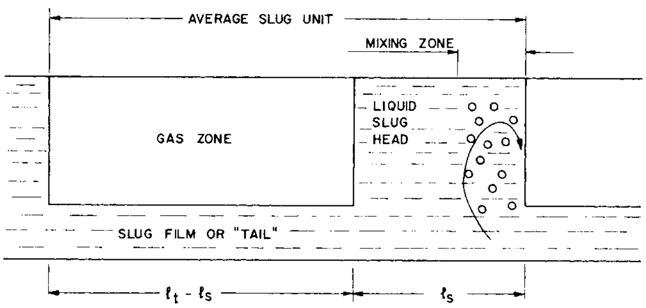


Fig. 2. Idealized slug unit.

section static pressure for runs with and without polymer, and as will be shown later, $\Delta P_{LE}/l_t$ was usually larger than the frictional component of the pressure gradient and thus mathematical accuracy was difficult to achieve.

In view of the above comments, the following approach was adopted.

1. For the same experimental mass fluxes of gas and liquid, the two-phase pressure gradient, slug velocity, and slug frequency were determined for experimental runs with and without polymer solution.
2. It was assumed that for low concentrations of polymer $\Delta P_{LE}/l_t$ would remain essentially constant for the same flow conditions with and without polymer.
3. The liquid slug hold-up R_s and the slug lengths l_s and l_t were obtained from the instantaneous void fraction data given in (10).

With this experimental data available, it was possible to solve Equation (3) for $\Delta P_{LE}/l_t$. The results can be presented in terms of Equation (5), the ratio of the lost energy to the frictional component of the pressure gradient.

$$\frac{\frac{\Delta P_{LE}}{l_t}}{\frac{\Delta P_f}{l_t}} = \frac{\frac{\Delta P_{LE}}{l_t}}{\frac{2 f_s\, \rho_L V_{ns}^2 R_s l_s}{D g_c\, l_t}} \tag{5}$$

Lockhart-Martinelli Pressure Loss Correlation

In addition to the above analysis, the two-phase pressure gradient data obtained in this study have been compared with the empirical Lockhart-Martinelli model (12). With this model an empirical relationship is given between ϕ_L and X, the Lockhart-Martinelli parameters, defined as

$$\phi_L^2 = \frac{\left(\frac{\Delta P}{\Delta L}\right)_{2\phi}}{\left(\frac{\Delta P}{\Delta L}\right)_{1\phi\text{Liquid}}} \tag{6}$$

and

$$X^2 = \frac{\left(\frac{\Delta P}{\Delta L}\right)_{1\phi\text{Liquid}}}{\left(\frac{\Delta P}{\Delta L}\right)_{1\phi\text{Gas}}} \tag{7}$$

The single phase pressure terms in the above equations are to be evaluated at the superficial velocities of the respective phases under the same rheological condition.

It was not anticipated that the present data would agree with one of the four original empirical curves, but it has been found that the Lockhart-Martinelli quantities ϕ and X are useful correlating parameters for a wide variety of two phase systems when properly defined, and might be similarly useful in the two phase drag reducing case.

Universal Drag Reduction Pressure Loss Correlation

To analyze the drag reduction data in both one phase and two phase flow, an attempt was also made to use the "universal drag reduction" curve of Virk (13), [as later modified by Arunachalam (2)]. The following generalization of the method of Virk was assumed to apply to both one- and two-phase flows:

1. Plot the drag reduction factor R defined as

$$R = \frac{1}{C}\left[1 - \frac{\left(\frac{\Delta P}{\Delta L}\right)_{wp}}{\left(\frac{\Delta P}{\Delta L}\right)_{np}}\right] \tag{8}$$

versus the polymer concentration C in ppm for fixed conditions of flow rate and gas/liquid ratio.

2. Determine tangents to the data plot as $C \to \infty$ and as $C \to 0$ and extrapolate to intersect at $[C]$, the intrinsic concentration. The value of R as $C \to 0$ approximates the intrinsic drag reduction $[R]$.

3. $[R]$ and $[C]$ are then used to normalize the R versus C data in terms of the universal drag reduction parameters γ and δ defined below:

$$\gamma = \frac{C}{[C]} \tag{9}$$

and

$$\delta = \frac{R}{[R]} \tag{10}$$

The original relationship between δ and γ as given by Virk (*13*) was

$$\delta = \frac{1}{1+\gamma} \tag{11}$$

Recently, Arunachalam (*2*) has demonstrated for three different polymer solutions over a wide range of flow rates and tube sizes that

$$\delta \simeq 1.2 \frac{1}{1+\gamma}, \ (\gamma \geq 0.2); \quad \delta \simeq 1, \ (0 \leq \gamma \leq 0.2)^{*} \tag{12}$$

The discrepancy between Equations (11) and (12) appears to have been caused by the degradation of solutions used by Virk.

EXPERIMENTAL RESULTS AND DISCUSSION

In any study involving dilute polymer solutions, degradation due to aging and shear are highly important. For this reason, extreme care was taken in the preparation of the solutions. A concentrated master solution was produced which was then diluted to the required concentration. Initially, it was hoped to recirculate the liquid phase, however, continuous shearing of the solutions appeared to indicate some degradation as shown in Figure 3 for 92 and 184 ppm solutions. For this reason once-through operation was selected, and reproducibility of pressure drop measurements with this precaution was satisfactory (within 5%).

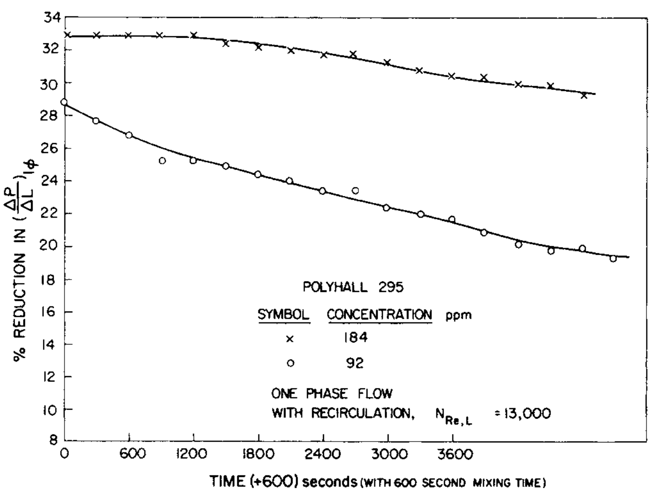


Fig. 3. Degradation rate of Polyhall 295.

* Since $\delta \to 1$ for $\gamma \to 0$ by definition.

One-Phase Flow

As a first step, the drag reduction characteristics of Polyhall 295 in one-phase turbulent flow of water were studied. Five different concentrations of Polyhall were used ranging from 10.75 to 184 ppm. The results are presented in Figure 4 in terms of the standard Fanning friction factor f versus N_{Re} plot.* At low concentrations (10.75 ppm) there is an appreciable drag reduction effect which increases with polymer concentration, appearing to reach a maximum at about 90 ppm as shown by the coincidence of the data for 92 and 184 ppm solutions. A concentration between 46 and 92 ppm appears to exist also where maximum drag reduction occurs per unit of polymer concentration. This observation is consistent with those made earlier by Fabula (*14*), Arunachalam (*2*), and Rodriguez et al. (*15*). A linear interpolation among the intercepts of the various lines shown in Figure 4 indicates that for Polyhall 295 the optimum concentration is 68 ppm.

Previous workers in the field have assumed that at the low concentrations involved, the fluids were essentially Newtonian (even though in more concentrated solutions non-Newtonian behavior may be observed). In order to verify this assumption, a part of this study was devoted to determining the rheology of these dilute solutions using a Weissenburg Rheogoniometer. This attempt turned out to present a formidable problem. Very high shear rates were necessary in order to get numerical values of strain rate of high enough magnitude to reduce the importance of experimental errors. The results are shown in Figure 5 as a plot of shear stress versus strain rate. It is evident that except for the 10.75 ppm solution, all others have a defi-

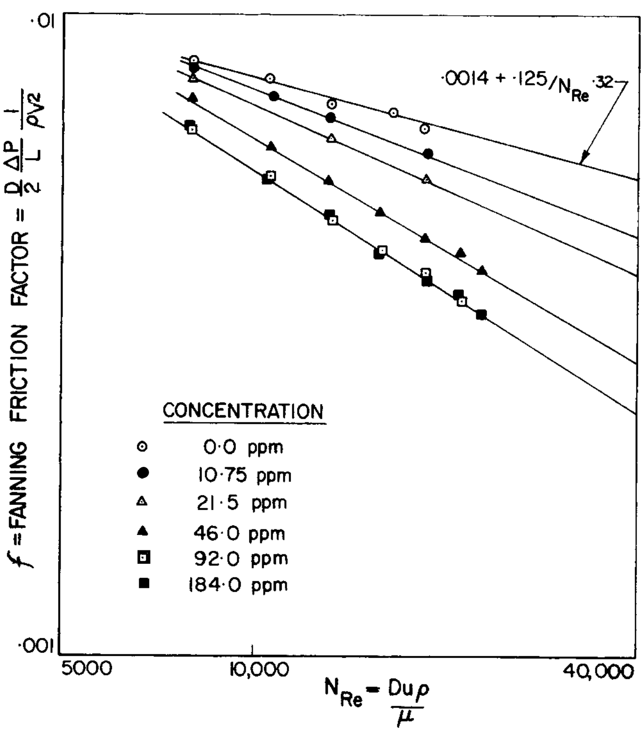


Fig. 4. Friction factor for single-phase liquid flow with polymer added.

* Viscosity of water was used in calculation of N_{Re}. This approach is consistent with other drag reduction data in the literature.

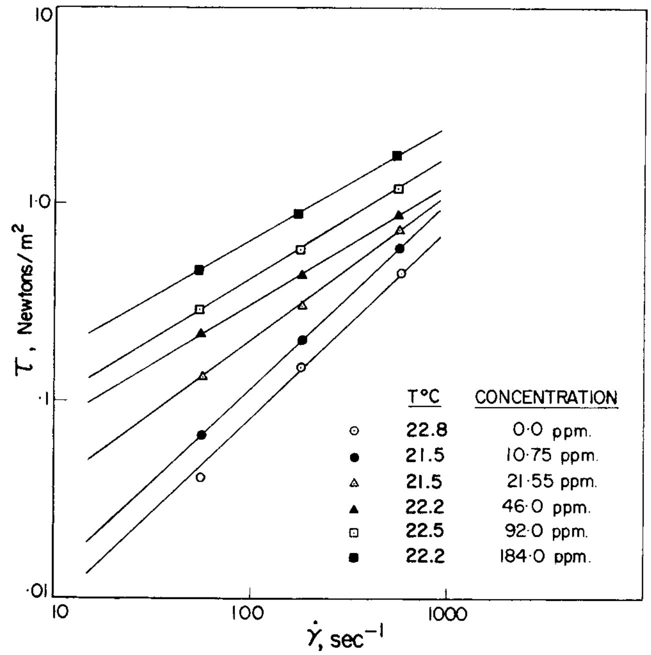


Fig. 5. Shear rate versus strain rate for Polyhall 295.

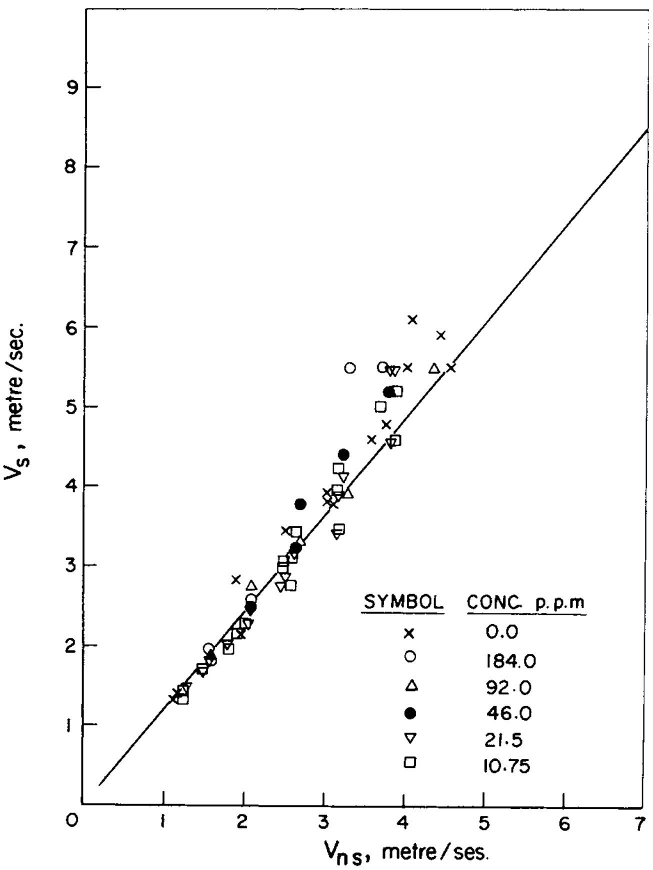


Fig. 6. Correlation of slug velocity with no-slip velocity for dilute polymer solutions.

nite pseudo-plastic non-Newtonian behavior. The constants K (the flow consistency index) and n, (the flow behavior index) for the polymer concentrations studied are given in Table 1 below.

This single phase drag reduction data was used to predict the drag reduction effect on the liquid slug head in two-phase slug flow.

Two-Phase Flow

The measured slug translational velocity versus the slug no-slip velocity calculated from measured input flow rates for air-water and air-Polyhall data is given in Figure 6. The results indicate that at lower values of the no-slip velocity, the slug translational velocity is unchanged by the presence of dilute polymer. The slope of the best fit line through the data of Figure 6 is 1.26, in agreement with the slope of 1.2 found by Gregory and Scott (*16*).

Similar effects are shown for the slug frequency data given in Figure 7. At low polymer concentrations, the slug frequency is the same as for the air-water system. At higher polymer concentrations, the fluid viscosity increases, and consequently the slug frequency decreases.

From the results of Figures 6 and 7, it is safe to conclude that the basic flow structure of the periodic slugs has not been disrupted by the presence of low concentrations of polymer in the liquid phase. Slug head velocity and slug frequency can be assumed to be independent of polymer concentration over a considerable range of concentrations and flow conditions.

Figure 8 represents a plot of the ratio of pressure gradients with and without polymer addition versus the superficial gas phase Reynolds number. The results show that the amount of reduction of the ratio increases with increasing gas velocity (which means also with increasing slug velocity), and this supports the suggestion made previously concerning the importance of the liquid slug head, because of its significantly higher velocity as the dominant factor in the axial pressure gradient. As in the case of single phase flow, over most of the gas velocity range, there appears to be an optimal polymer concentration for maximum unit drag reduction. The reduction in two-phase flow is always substantially larger than for the single phase case at the same superficial liquid velocity, due to

Table 1. Values of "K" and "n" for Dilute Polymer Solutions

Concentration ppm (in water)	$\frac{K \text{ Dynes (Sec}^n)}{\text{cm}^2} = (10)^{-1} \frac{\text{N.S.}^n}{\text{m}^2}$	n
0.0	0.0095	1.04
10.75	0.015	0.978
21.5	0.070	0.750
46.0	0.192	0.614
92.0	0.266	0.619
184.0	0.520	0.584

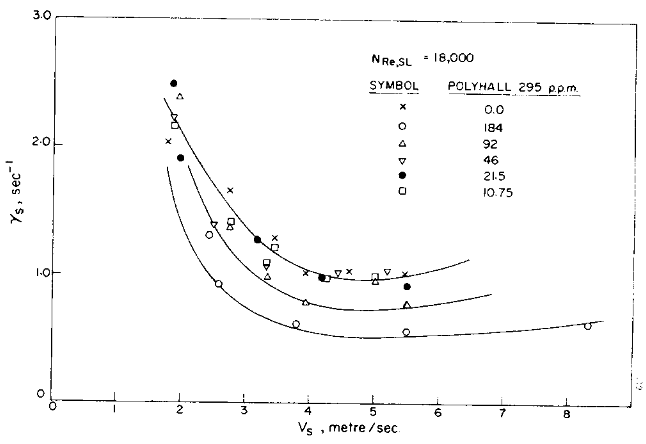


Fig. 7. Slug frequency as a function of slug velocity for dilute polymer solutions.

the velocity effect on drag reduction. The faster the slug moves the more the drag reduction, as shown in Figure 8.

Data on slug structure, slug frequency, and slug velocities for runs with and without polymer was used to calculate $\Delta P_{LE}/l_t$ from Equation (3). Results of these calculations indicated that the lost energy term $\Delta P_{LE}/l_t$ was in fact greater than the frictional component for most flow conditions. Figure 9 shows a plot of the ratio of the lost energy component to the frictional component of the axial pressure gradient versus the slug no-slip velocity for both

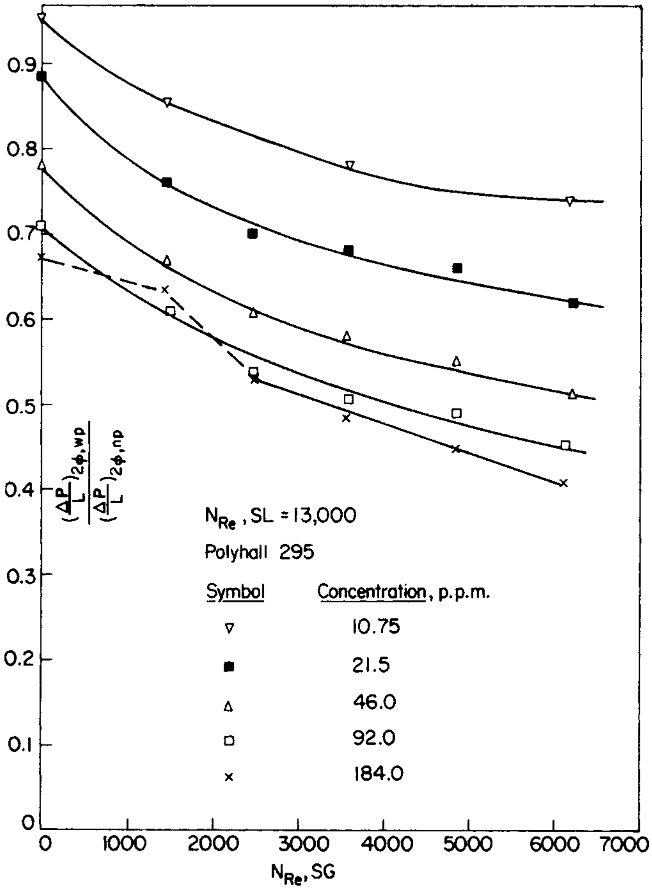


Fig. 8. Typical two-phase drag reduction results.

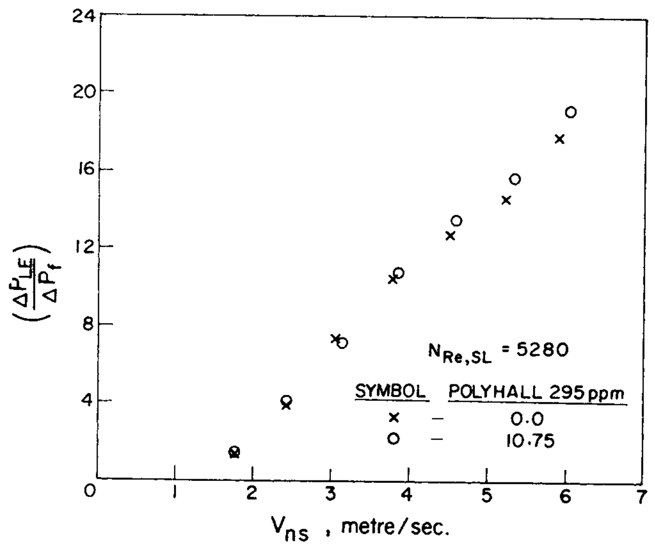


Fig. 9. Ratio of accelerational energy loss to frictional energy loss as a function of no slip velocity, for a superficial liquid Reynolds number of 5280.

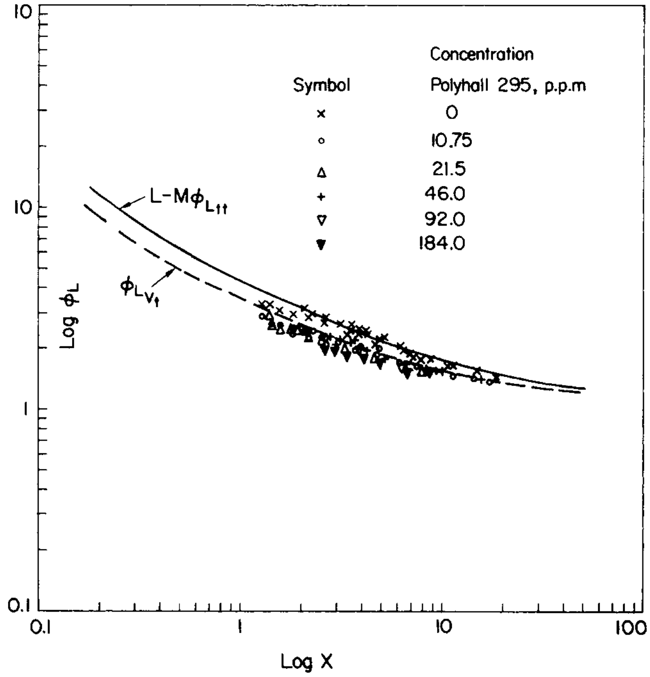


Fig. 10. Pressure drop correlation for dilute polymer solutions using Lockhart-Martinelli parameters.

air-water and air-10.75 ppm Polyhall, for a superficial liquid Reynolds number of 5280. The results indicate that as the gas velocity increases, the importance of the lost energy term increases. Also, for most cases, the lost energy term is much greater than the frictional component. The data for the Polyhall generally fall higher than that for water only due to a smaller denominator (ΔP_f) for a nearly constant numerator (ΔP_{LE}). The assumption made earlier of the lost energy term, $\Delta P_{LE}/l_t$ being constant for a given flow condition will apply only to the lower polymer concentrations. At higher flow rates and concentrations, it is probable that the polymer rheology will have some effect on the magnitude of the lost energy component.

Figure 10 presents a plot of drag reduction data for several polymer concentrations analyzed in terms of the Lockhart-Martinelli model. It is significant to note that the data for polymer solutions falls below the corresponding correlation for water, however, it does cluster near the ϕ_{Lvt} line (i.e., viscous liquid, turbulent gas).

The only previous study on non-Newtonian two-phase flow, that of Oliver and Young Hoon (*17*), observed phenomena similar to the results in Figure 10. That is, the Lockhart-Martinelli approach although empirical correlates the two-phase drag reduction data but at lower ϕ_L values than predicted by Lockhart-Martinelli.

The single and two-phase drag reduction data were also analyzed in terms of the "universal drag reduction curve." Figure 11 presents data in terms of the parameters of Equations (11) and (12) for three different single phase liquid Reynolds numbers, and five different two-phase flow conditions for five different polymer concentrations. An unusual degree of correlation of the single phase and two phase systems is evident. The data in general falls above the line $\delta = \dfrac{1}{1+\gamma}$, [Equation (11)] and gives more support to the conclusion of Arunchalam (*2*) that $\delta = \dfrac{1.2}{1+\gamma}$ [Equation (12)]. The data scatter is similar to that observed by Virk (*13*) and Arunachalam (*2*).

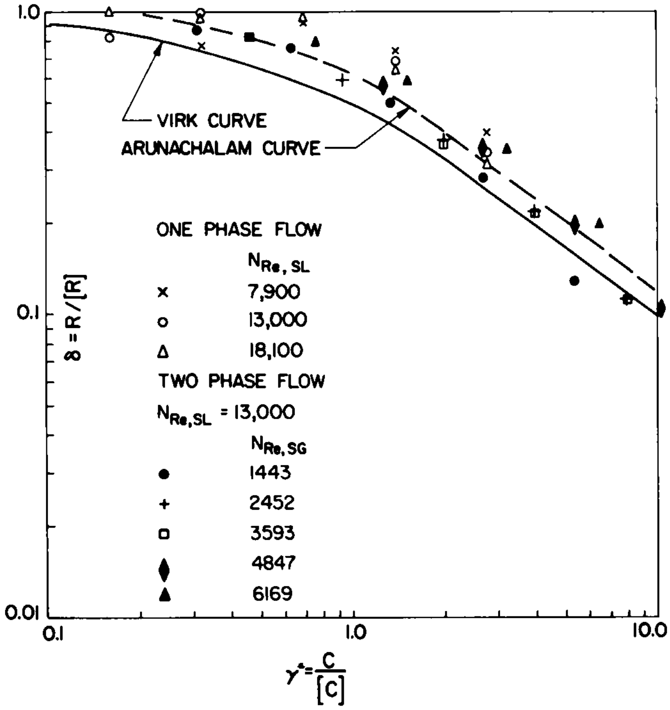


Fig. 11. Universal drag reduction curve correlation.

CONCLUSIONS

1. Drag reduction phenomena are observed in concurrent gas-liquid flow. The degree of reduction in the axial pressure gradient is always greater than in the single phase case at the same superficial liquid Reynolds number.
2. Drag reduction studies on slug flow have shown the importance of the accelerational term in the axial pressure gradient. This energy term in slug flow exceeds the frictional component of the axial pressure gradient. The use of a Toms phenomenon fluid for determination of the magnitude of these accelerational effects is believed to be unique, and the method could be extended to other two phase flow patterns where accelerational effects may be significant.
3. The empirical Lockhart-Martinelli parameters ϕ_L and X will correlate the drag reduction data, but the data falls approximately 20% below the corresponding Lockhart-Martinelli line.
4. Both the single- and two-phase data obtained from this study can be correlated by the same universal drag reduction curve of the type proposed by Virk.
5. Data are presented for slug frequencies and slug velocities in two-phase slug flow with and without polymer solutions, which show that the polymer additions do not affect the periodic slugging behavior over considerable flow ranges and in the more dilute polymer concentration ranges.

ACKNOWLEDGMENT

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NOTATION

C = polymer concentration in water, ppm
(C) = intrinsic drag reduction concentration, ppm
D = tube diameter, m
f = Fanning friction factor
g_c = universal gravity constant
K = flow consistency index, dynes/cm^2 (sec)^n = (10)$^{-1}$ N.S.^{n}/m^2
l = length, m
n = flow behavior index
N_{Re} = Reynolds number
ΔP = pressure drop, N/m^2
R = drag reduction factor, ppm^{-1}
(R) = intrinsic drag reduction factor, ppm^{-1}
R_s = slug head liquid hold-up
V = velocity, m/S
ρ = density, kg/m^3
μ_L = liquid viscosity, N.S./m^2
γ_s = slug frequency, s^{-1}
δ and γ = universal drag reduction parameters, Equations (9) and (10)
ϕ and X = Lockhart-Martinelli parameters, Equations (6) and (7)
τ = shear stress, N/m^2
$\dot{\gamma}$ = strain rate s^{-1}

Subscripts

f = frictional
LE = lost energy or acceleration
L = liquid phase
ns = no-slip
np = no polymer
S = slug
t = total
2ϕ = two-phase
1ϕ = one-phase
wp = with polymer
SG = superficial gas
SL = superficial liquid

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